AVAILABILITY AND VARIOUS STATISTICAL MEASURES OF TWO UNITS REDUNDANT SYSTEM WITH THREE TYPES OF FAILURES UNDER WAITING TIME TO REPAIR WITH COMMON CAUSE FAILURES

By

DIGVIJAY SINGH — **Deptt.Of Mathematics** — **J.P.Engg.College,Meerut,India.** — **drdigvijay2008@rediffmail.com**

VIRESH SHARMA — **Deptt.Of Mathematics** — **N.A.S.(P.G.) College,Meerut,India.** — **vireshsharma1@yahoo.com**

SMITA SHARMA — **Deptt. Of Statistics** — **D.A.VCollege,Dehradun,India** — **smita.sharma73@yahoo.in**

ABSTRACT : In this paper, the author has considered a transient system composed of two identical units in a standby mode, which can fail due to hardware and common cause failure. Initially, one unit is in operational mode and other is in standby mode. The operator unit may fail partially or totally. In case of total failure of first unit, standby unit becomes operational and system works with full efficiency.

KEYWORDS: Repairable system, supplementary variables standby system.

INTRODUCTION: The author has considered a transient system composed of two identical units in a standby mode, which can fail due to hardware and common cause failure. Initially, one unit is in operational mode and other is in standby mode. The operator unit may fail partially or totally. In case of total failure of first unit, standby unit becomes operational and system works with full efficiency. It is observed that whenever there is a hardware failure, system goes to partial failure mode first and then to total failure mode when there is common cause failure. Failure time distribution are exponential and repairs are general.

NOTATIONS:

S_o, S_2 : Normally operating state,

S_1, S_3 : Degraded state,

S_4, S_5 : Failed state,

$$D/Dt/Dx/Dy/Dz/Dw: \frac{d}{dt}/\frac{\partial}{\partial t}/\frac{\partial}{\partial x}/\frac{\partial}{\partial y}/\frac{\partial}{\partial z}/\frac{\partial}{\partial w}$$
,

$\alpha_1/\alpha_2/\alpha_3/\alpha_4$: Constant failure rate due to minor/major error,

λ_c : Constant failure due to common cause failure,

$P_{0s}t$:P(at time t the system is in state S_0),

$P_{Ps}t(x,t)\Delta : P$: P (the system is in state S_1 at time t due to minor failure and repair time lies in the interval $(x, x+\Delta)$,

$P_{T0}t(y,t)\Delta$: P (the system is in state S_2 at time t and elapsed repair time lies in the interval $(y, y+\Delta)$,

$P_{TP}t(z,t)\Delta$: P (the system is in the state S_3 at time t and elapsed repair time lies in interval $(z, z+\Delta)$,

$P_{TT}t(w,t)\Delta$: P(the system is state S_4 at time t and elapsed repair time lies in the interval $(w, w+\Delta)$,

$P_C(x,t)\Delta$: P(the system is in state S_5 at time t and elapsed repair time lies in interval $(x, x+\Delta)$, , $S_i^K(s) = K(x)e^{-\int_0^x Kxdx}$

where

$$K = \beta_1, \psi_1, \phi_1, \phi_2, w_c \quad , \quad \sum_i = \sum_{i=1}^{n} \ ,$$

$\int = \int_o^\infty$, unless otherwise stated.

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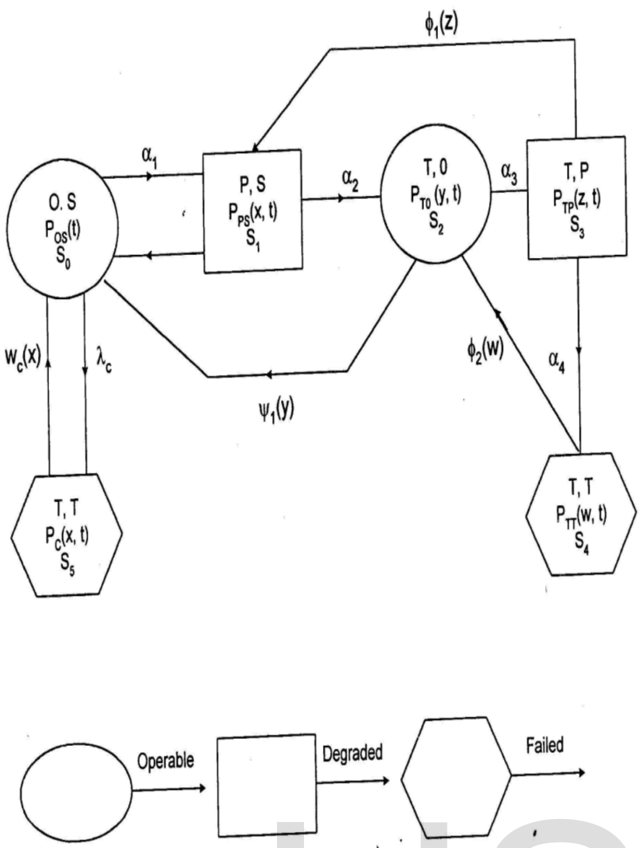

Transition Diagram

Formulation of the Model : Viewing the nature of this problem the following set of difference differential equations is obtained.

$$\left(D+\alpha_1+\lambda_C\right)P_{os}^{(t)}=\int\beta_1(x)P_{PS}(x,t)\,dx$$

$$+\int w_C(x)P_C(x)\,dx+\int\psi_1(y)P_{T0}(y,t)\,dy, \quad (1)$$

$$\left[Dx+Dt+\alpha_2+\beta_1(x)\right]P_{PS}(x,t)=0, \quad (2)$$

$$\left[Dy+Dt+\alpha_3+\psi_1(y)\right]P_{T0}(y,t)=0, \quad (3)$$

$$\left[Dz+Dt+\alpha_4+\phi_1(z)\right]P_{TP}(z,t)=0, \quad (4)$$

$$\left[Dw+Dt+\phi_2(w)\right]P_{TT}(w,t)=0, \quad (5)$$

$$\left[Dx+Dt+w_C(x)\right]P_C(x,t)=0. \quad (6)$$

BOUNDARY CONDITIONS :

$$P_{PS}(0,t)=\alpha_1P_{0s}(t)+\int\phi_1(z)P_{TP}(z,t)\,dz, \quad (7)$$

$$P_{T0}(0,t)=\alpha_2\int P_{Ps}(x,t)\,dt+\int\phi_2(w)P_{TT}(w,t)\,dw, \quad (8)$$

$$P_{TP}(0,t)=\alpha_3\int P_{T0}(y,t)\,dy, \quad (9)$$

$$P_{TT}(0,t)=\alpha_4\int P_{T0}(z,t)\,dz, \quad (10)$$

$$P_C(0,t)=\lambda_CP_{0s}(t). \quad (11)$$

Initial conditions :

$P_{0S}(0)=1$, otherwise 0.

Solution of the Model :

$$\left(s+\alpha_1+\lambda_C\right)P_{0S}^*(s)=1+\int\beta_1(x)P_{PS}^*(x,s)\,dx$$

$$+\int w_C(x)P_C^*(x,s)\,dx+\int\phi_1(y)P_{T0}(y,s)\,dy, \quad (12)$$

$$\left(s+\alpha_2+\beta_1(x)+\frac{\partial}{\partial x}\right)P_{PS}^*(x,s)=0, \quad (13)$$

$$\left(s+\frac{\partial}{\partial y}+\alpha_3+\psi_1(y)\right)P_{T0}^*(y,s)=0, \quad (14)$$

$$\left(s+\frac{\partial}{\partial z}+\alpha_4+\phi_1(z)\right)P_{TP}^*(z,s)=0, \quad (15)$$

$$\left(s+\frac{\partial}{\partial w}+\phi_2(w)\right)P_{TT}^*(w,s)=0, \quad (16)$$

$$\left(s+\frac{\partial}{\partial x}+w_C(x)\right)P_C^*(x,s)=0. \quad (17)$$

$$P^*_{Ps}(0,s) = \alpha_1 P^*_{0s}(s) + \int \phi_1(z) P^*_{TP}(z,s)\,dz, \tag{18}$$

$$P^*_{T0}(0,s) = \alpha_2 \int P^*_{Ps}(x,s)\,ds + \int \phi_2(w) P^*_{TT}(w,s)\,dw, \tag{19}$$

$$P^*_{TP}(0,s) = \alpha_3 \int P^*_{T0}(y,s)\,dy, \tag{20}$$

$$P^*_{TT}(0,s) = \alpha_4 \int P^*_{TP}(z,s)\,dz, \tag{21}$$

$$P^*_{C}(0,s) = \lambda_C P^*_{0s}(s). \tag{22}$$

$$\bar{P}_{0s}(s) = \frac{1}{D(s)}, \tag{23}$$

$$\bar{P}_{PS}(s) = \frac{B(s)}{D(s)}, \tag{24}$$

$$\bar{P}_{T0}(s) = \frac{1}{A(s)}\alpha_1\alpha_2.\frac{1-\bar{S}_{\beta_1}(s+\alpha_2)}{S+\alpha_2}.\frac{1-S\psi_1(s+\alpha_3)}{s\beta+\alpha_3}.\frac{1}{D(s)}, \tag{25}$$

$$\bar{P}_{TP}(s) = \frac{C(s)}{D(s)}, \tag{26}$$

$$\bar{P}_{TT}(s) = \alpha_4 \frac{1-\bar{S}_{\phi_2}(s)}{s}\frac{C(s)}{D(s)}, \tag{27}$$

$$\bar{P}_{C}(s) = \lambda_C \frac{1-\bar{S}_{w_c}(s)}{s}\frac{1}{D(s)}. \tag{28}$$

Evaluation of Laplace transforms of up and down state Probabilities.

$$\bar{P}_{up}(s) = \bar{P}_{0S} + \bar{P}_{PS}(s) + \bar{P}_{T0}(s) + \bar{P}_{TP}(s)$$

$$= \left[1+\beta(s)+\frac{\alpha_1\alpha_2}{A(s)}.\frac{1-\bar{S}_{\beta_1}(s+\alpha_2)}{s+\alpha_2}.\frac{1-\bar{S}_{\psi_1}(s+\alpha_3)}{s+\alpha_3}+C(s)\right]\frac{1}{D(s)}, \tag{29}$$

$$\bar{P}_{down}(s) = \bar{P}_{TT}(s) + \bar{P}_{C}(s),$$

$$= \left[\alpha_4 C(s)\frac{1-\bar{S}_{\phi_2}(s)}{s} + \lambda_C \frac{1-\bar{S}_{wc}(s)}{s}\right]\frac{1}{D(s)}. \tag{30}$$

It is worth noticing that

$$\bar{P}_{up}(s) + \bar{P}_{down}(s) = \frac{1}{s}$$

ERGODIC BEHAVIOUR OF THE SYSTEM:

Using Abel's Lemma in Laplace transforms viz.,

$$\lim_{s\to 0} S\bar{f}(s) = \lim_{t\to\infty} f(t) = f,(say)$$

$$P_{up} = \underset{s\to 0}{s} P^*_{up}(s)$$

$$= s\left[1+\beta(s)+\frac{\alpha_1\alpha_2}{A(s)}.\frac{1-\bar{S}_{\beta_1}(s+\alpha_2)}{s+\alpha_2}.\frac{1-\bar{S}_{\psi_1}(s+\alpha_3)}{s+\alpha_3}+C(s)\right]\frac{1}{D(s)} \tag{31}$$

$$P_{down} = \underset{s\to 0}{} \frac{1}{D(s)}\left[\alpha_4(s)^{1-\bar{S}_{\phi 2}(s)} + \lambda_C^{\,1-\bar{S}_{wC}(s)}\right] \tag{32}$$

$$P_{up} = \left[1+\beta(s)+\frac{\alpha_1\alpha_2}{A(0)}.\frac{1-\bar{S}_{\beta_1}(s+\alpha_2)}{\alpha_2}.\frac{1-\bar{S}_{\psi_1}(s+\alpha_3)}{\alpha_3}+C(0)\right]\frac{1}{D'(0)} \tag{33}$$

$$= \left[1+B(0)+C(0)\frac{\alpha_1\alpha_2}{A(0)(\beta_1+\alpha_2)(\psi_1+\alpha_3)}\right]\frac{1}{D'(0)}. \tag{34}$$

$$P_{down} = \left[\alpha_4 C(0) M^{\phi_2} + \lambda_C M^{w_c}\right]\frac{1}{D'(0)}. \tag{35}$$

It is obvious that

$$P_{up} + P_{down} = 1, \tag{36}$$

where,
$$D'(0) = \frac{d}{ds}\left(D(s)\right) = {}_{s\to 0}\ \frac{D(s)}{s}, \tag{37}$$

and
$$M^K\left(K = \phi_2, w_c\right) = -\bar{S}_i^{'K}(0). \tag{38}$$

PARTICULAR CASE :

When repair time follows exponential time distributions :

$$\bar{S}^{\theta}(s)\frac{\theta}{(s+\theta)} \tag{39}$$

Where $\theta = \psi_1, \phi_1, \beta_1, \phi_2, w_c$ in relation above, we find

$$\bar{P}_{os}(s) = \frac{1}{E(s)}, \tag{40}$$

$$\bar{P}_{Ps}(s) = \frac{F(s)}{E(s)}, \tag{41}$$

$$\bar{P}_{T0}(s) = \frac{1}{G(s)}\alpha_1\alpha_2\frac{1}{s+\alpha_2+\beta_1}\frac{1}{s+\alpha_3+\psi_1}.\frac{1}{E(s)}, \tag{42}$$

$$\bar{P}_{TP}(s) = \frac{H(s)}{E(s)}, \tag{43}$$

$$\bar{P}_{TT}(s) = \alpha_4\frac{1}{s+\phi_2}\frac{H(s)}{E(s)}, \tag{44}$$

$$\bar{P}_C(s) = \frac{\lambda_C}{s+w_C}\frac{1}{E(s)}. \tag{45}$$

COST FUNCTION ANALYSIS OF THE SYSTEM :

The s-expected up time of the system dunring]0,t] is given by

$$E(t) = \int_0^t P_{up}(t)\,dt$$

and s-expected busy period of the service in]0,t] is

$$\mu_B(t) = t - \int_0^t P_{down}(t)\,dt.$$

Hence, net expected gain G(t) is as follows

$$G(t) = C_1\int_0^t P_{up}(t)\,dt - C_2\left[t - \int_0^t P_{down}(t)\,dt\right]$$

$$= C_1\left\{\frac{0.3043}{0.7}e^{-0.7t} + \frac{3.173}{1.2}e^{-1.2t} - \frac{7.3636}{1.25}e^{-1.25t} + \frac{2.51}{1.35}e^{-1.35t}\right\}_0^t$$

$$-C_2\left[t - \left\{1.0038t - \frac{1.0637}{0.7}e^{-0.7t} + \frac{75.2}{1.2}e^{-1.2t} - \frac{30.69}{1.35}e^{-1.35t} - \frac{102.34}{1.25}e^{-1.25t}\right\}\right]_0^t$$

$$G(t) = C_1\left[-0.4347e^{-0.7} + 2.644e^{-1.2t} - 5.891e^{-1.25t} + 1.859e^{-1.35t} + 1.576\right]$$

$$-C_2\left[t + 1.51957e^{-0.7} - 62.66e^{-1.2t} - 22.73e^{-1.35t}\right.$$

$$\left. + 1.187e^{-1.25t} - 0.0038t - 36.598\right].$$

Here, C_1 and C_2 are the revenue per unit of time and repair cost per unit time (spent in service facility), respectively.

NUMERICAL COMPUTATIONS :

Setting t = 0, 1, 2, 3, 4, 5, we have following values given in table.

S.NO.	Time(t)	P up (t)	P down (t)
1	0	1.9849	-0.9825
2	1	0.65455	0.3340
3	2	0.22289	0.7761
4	3	0.08001	0.92033
5	4	0.02066	1.0002
6	5	0.0113	1.008

Similarly, setting $C_1 = 1, C_2 = 0.5$

Interpretation of Results

Table (1) computes the availability of the system at an instant 't' and shows the availability versus time. The availability very slowly offers a long time showing that the system remains available for a long period of time

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